UPDATES FOR "NUMBER THEORY AND GEOMETRY"

Dear Readers,

Here is a list of updates (known typos, errors and omissions, expanded paragraphs, etc.), with the text as it appeared in the first edition, followed by the corrected text (as it will/should appear in a revised edition).

Acknowledgements: I'd like to thank Keith Conrad, Giacomo De Leva, Fernando Gouvêa, Hanson Smith, and Cindy Zhang for identifying a number of typos and errors listed below.

(1) **Page 11. Example 1.4.1.** The line L is $y = x + 1$.
- (**Old text**) With a little bit of basic plane geometry, we find an equation for $L : y = x - 1$ (see Exercise 1.8.8).
- (**New text**) With a little bit of basic plane geometry, we find an equation for $L : y = x + 1$ (see Exercise 1.8.8).

(2) **Page 24. Exercise 1.8.6.** In part (3), it should specify that the result is for a non-zero complex number α.
- (**Old text**) Show that any complex number α can be written uniquely as...
- (**New text**) Show that any non-zero complex number α can be written uniquely as...

(3) **Page 38. Proof of Theorem 2.3.14.** There are a couple of issues with the second paragraph of the proof, which should read like this instead:
- (**New text**) Next, let us show the induction step. Let us assume that the theorem is true for sets with more than k elements, and let S be a set with more than $k + 1$ elements. Let $S_1, S_2, \ldots, S_k, S_{k+1}$ be $k + 1$ subsets of S, such that $\bigcup_{i=1}^{k+1} S_i = S$. If the set S_{k+1} has more than one element, then we are done. Otherwise, suppose that S_{k+1} contains only one element of S, or $S_{k+1} = \emptyset$. Then, the union $S' = \bigcup_{i=1}^{k} S_i$ contains at least $k + 1$ elements (since S contains $k + 2$ elements, and the missing subset S_{k+1} contains ≤ 1 elements, so S' contains $\geq k + 2 - 1 = k + 1$ elements). Hence, using our induction hypothesis on the set S', we conclude that one of $S_1, \ldots, S_k$ contains at least two elements.

(4) **Page 40. Line 5**
- (**Old text**) Since $r \in S$, there must be...
- (**New text**) Since $r_1 \in S$, there must be...

(5) **Page 71. Theorem 3.3.11**
- (**Old text**) Let $m > 0$ and $a \geq 0$ be fixed integers...

- (**New text**) Let $m > 0$ and $a > 0$ be fixed integers...

(6) **Page 74. Conjecture 3.4.7** The statement should be for $k \geq 1$.
- (**Old text**) Let $k \geq 0$ be an integer,...

- (**New text**) Let $k \geq 1$ be an integer,...

(7) **Page 75. Conjecture 3.4.10.** The statement of the Bateman-Horn conjecture needs two additional assumptions:
- Each factor $f_i(x)$ should have a positive leading coefficient, i.e., if $f_i(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, then $a_n \geq 1$.
- The values of $f(x)$ on $\mathbb{Z}$ must not all share a common prime factor.

(8) **Page 81. Exercise 3.5.22**
- (**Old text**) Show that if $a > 0$ and...

- (**New text**) Show that if $a > 1$ and ...

(9) **Page 114. Exercise 4.7.26**
- (**Old text**) ...let $k \in \mathbb{Z}$ such that $1 \leq k \leq p$...

- (**New text**) ...let $k \in \mathbb{Z}$ such that $1 \leq k < p$...

(10) **Page 150. Exercise 5.6.6 part (3)**
- (**Old text**) (Hint: if $a * H$ and $b * H$ are two...

- (**New text**) (Hint: if $g_1 * H$ and $g_2 * H$ are two...

(11) **Page 231. Exercise 8.10.10 part (b)**
- (**Old text**) ..., then $p = 2$ and $a = 1$.

- (**New text**) ..., then $p = 2$ and $a \equiv 1 \bmod 2$.

(12) **Page 388. Corollary 13.3.4.** In part (2) of the statement, d should be t.
- (**Old text**) If $|\alpha - s/t| < |\alpha - p_k/q_k|$ for some $k \geq 1$, then $d > q_k$.

- (**New text**) If $|\alpha - s/t| < |\alpha - p_k/q_k|$ for some $k \geq 1$, then $t > q_k$.

(13) **Page 391. Exercise 13.4.14.** Here e should also be d, because otherwise $\overline{\alpha + \beta}$ is not defined with the definition we have given for conjugation in the statement.
- (**Old text**) ... $\alpha = u + v\sqrt{d}$ and $\beta = x + y\sqrt{e}$, where $u, v, x, y \in \mathbb{Q}$ and d, e are non-zero integers that are not perfect squares.

- (**New text**) ... $\alpha = u + v\sqrt{d}$ and $\beta = x + y\sqrt{d}$, where $u, v, x, y \in \mathbb{Q}$.

(14) **Page 391. Exercise 13.4.20.** Here d should be $n^2 + 2n$.

- (**Old text**) Let d be a positive integer such that $d = n^2+1$, for some integer $n > 1$.

- (**New text**) Let d be a positive integer such that $d = n^2 + 2n$, for some integer $n > 1$.

(15) **Page 407. Proof of 14.3.17.** A $+$ should be a $\cdot$ in the last line of the previous to last displayed equation, because they are elements living in the group $U = (U, \cdot)$ where the operation is multiplication.

- (**Old text**) $\psi((a \bmod 2, n)) + \psi((b \bmod 2, m))$.

- (**New text**) $\psi((a \bmod 2, n)) \cdot \psi((b \bmod 2, m))$.