

*Mathematics is the queen of the sciences and number theory is the queen of mathematics. (Die Mathematik ist die Königin der Wissenschaften und die Zahlentheorie ist die Königin der Mathematik.). - Carl Friedrich Gauss*

**Question 1.** Show that  $n$  and  $n + 1$  are coprime for all  $n \geq 1$ .

**Question 2.** Show that if  $e$  divides  $a$  and  $b$  then  $e$  divides  $ar + bs$  for any integers  $r$  and  $s$ .

**Question 3.** Use Euclid's algorithm to find the following GCD's:

(a)  $(121, 365)$ ,

(b)  $(89, 144)$ ,

(c)  $(295, 595)$ ,

(d)  $(1001, 1309)$ .

**Question 4.** Find the GCD of 17017 and 18900 using Euclid's algorithm.

**Question 5.** Find  $d$ , the GCD of  $a$  and  $b$ , i.e.,  $d = (a, b)$ , and  $r, s \in \mathbb{Z}$  such that  $ar + bs = d$ :

(a)  $a = 267$  and  $b = 112$ ,

(b)  $a = 242$  and  $b = 1870$ .

**Question 6.** Find all solutions with integer coefficients  $x$  and  $y$ :

(a)  $267x + 112y = 3$ ,

(b)  $376x + 72y = 18$ .

**Question 7.** Find all solutions with integer coefficients  $x$  and  $y$ :

(a)  $203x + 119y = 47, 48, \text{ or } 50$ ,

(b)  $203x + 119y = 49$ .

**Question 8.** Prove that if  $(a, b) = d$  then  $(\frac{a}{d}, \frac{b}{d}) = 1$ .

**Question 9.** Find all the natural, integral and rational roots of the polynomial equation

$$5x^3 + 27x^2 - 153x + 81 = 0.$$

**Question 10.** Show that if  $n$  is not prime then  $n$  has a prime divisor  $\leq \sqrt{n}$ .

**Question 11.** Is 44497 prime? Why, or why not?

**Question 12.**

(a) Prove that a natural number is a square if and only if the exponent of each prime factor is even.

(b) Prove that if a number  $n$  is not a square then  $\sqrt{n}$  is irrational.

**Question 13.** Show that  $100^{(1/3)}$  is irrational.

**Question 14.** Show that if  $a, b$  are natural numbers with  $(a, b) = 1$  and  $ab$  is a square, then  $a$  and  $b$  are also squares.