Question 1. Show that $n$ and $n + 1$ are coprime for all $n \geq 1$.

Question 2. Show that if $e$ divides $a$ and $b$ then $e$ divides $ar + bs$ for any integers $r$ and $s$.

Question 3. Use Euclid’s algorithm to find the following GCD’s:
   (a) $(121, 365)$,
   (b) $(89, 144)$,
   (c) $(295, 595)$,
   (d) $(1001, 1309)$.

Question 4. Find the GCD of 17017 and 18900 using Euclid’s algorithm.

Question 5. Find $d$, the GCD of $a$ and $b$, i.e., $d = (a, b)$, and $r, s \in \mathbb{Z}$ such that $ar + bs = d$:
   (a) $a = 267$ and $b = 112$,
   (b) $a = 242$ and $b = 1870$.

Question 6. Find all solutions with integer coefficients $x$ and $y$:
   (a) $267x + 112y = 3$,
   (b) $376x + 72y = 18$.

Question 7. Find all solutions with integer coefficients $x$ and $y$:
   (a) $203x + 119y = 47, 48, 50$,
   (b) $203x + 119y = 49$.

Question 8. Prove that if $(a, b) = d$ then $(\frac{a}{d}, \frac{b}{d}) = 1$.

Question 9. Find all the natural, integral and rational roots of the polynomial equation
   $$5x^3 + 27x^2 - 153x + 81 = 0.$$ 

Question 10. Show that if $n \geq 2$ is not prime then $n$ has a prime divisor $\leq \sqrt{n}$.

Question 11. Is 44497 prime? Why, or why not?

Question 12. 
   (a) Prove that a natural number is a square if and only if the exponent of each prime factor is even.
   
   (b) Prove that if a number $n$ is not a square then $\sqrt{n}$ is irrational.

Question 13. Show that $100^{1/3}$ is irrational.

Question 14. Show that if $a, b$ are natural numbers with $(a, b) = 1$ and $ab$ is a square, then $a$ and $b$ are also squares.