> *Mathematics is the queen of the sciences and number theory is the queen of mathematics. (Die Mathematik ist die Königin der Wissenschaften und die Zahlentheorie ist die Königin der Mathematik.).* - Carl Friedrich Gauss

Question 1. Show that n and $n+1$ are coprime for all $n \geq 1$.

Question 2. Show that if e divides a and b then e divides $ar + bs$ for any integers r and s.

Question 3. Use Euclid's algorithm to find the following GCD's:
(a) $(121, 365)$,

(b) $(89, 144)$,

(c) $(295, 595)$,

(d) $(1001, 1309)$.

Question 4. Find the GCD of 17017 and 18900 using Euclid's algorithm.

Question 5. Find d, the GCD of a and b, i.e., $d = (a, b)$, and $r, s \in \mathbb{Z}$ such that $ar + bs = d$:
(a) $a = 267$ and $b = 112$,

(b) $a = 242$ and $b = 1870$.

Question 6. Find all solutions with integer coefficients x and y:
(a) $267x + 112y = 3$,

(b) $376x + 72y = 18$.

Question 7. Find all solutions with integer coefficients x and y:
(a) $203x + 119y = 47$, 48, or 50,

(b) $203x + 119y = 49$.

Question 8. Prove that if $(a, b) = d$ then $(\frac{a}{d}, \frac{b}{d}) = 1$.

Question 9. Find all the natural, integral and rational roots of the polynomial equation

$$5x^3 + 27x^2 - 153x + 81 = 0.$$

Question 10. Show that if $n \geq 2$ is not prime then n has a prime divisor $\leq \sqrt{n}$.

Question 11. Is 44497 prime? Why, or why not?

Question 12.
(a) Prove that a natural number is a square if and only if the exponent of each prime factor is even.

(b) Prove that if a number n is not a square then $\sqrt{n}$ is irrational.

Question 13. Show that $100^{(1/3)}$ is irrational.

Question 14. Show that if a, b are natural numbers with $(a, b) = 1$ and ab is a square, then a and b are also squares.